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# DETERMINATION OF THE ORIENTATION OF THE AXIS OF A ROCKET OR SATELLITE IN ITS TRAJECTORY OR ORBIT

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AIR FORCE CAMBRIDGE RESEARCH LABORATORIES
OFFICE OF AEROSPACE RESEARCH
UNITED STATES AIR FORCE
BEDFORD, MASSACHUSETTS

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## ABSTRACT

A method is developed for determining the aspect of the axis of a rocket or satellite with respect to an earth based system of coordinates for the case where these bodies undergo a constant precessional motion about some fixed direction. The analysis is based on data obtained from a magnetometer mounted on the body so as to give the axial component of the Earth's magnetic field and a sun sensor which measures the angle between the Sun Vector and the axis of the body. The only information needed for the determination of the aspect of the axis is the maximum and minimum axial components of the Earth's magnetic field, the maximum and minimum angles between the axis of the body and the Sun Vector, the time sequence in which these maximums and minimums occur, and the angular velocity of precession.

## 1. ANGLE OF THE CONE OF PRECESSION; ANGLE BETWEEN AXIS OF PRECESSION, EARTH'S MAGNETIC FIELD $\beta_H$, AND SUN $\beta_S$.

We will assume in what follows that when the rocket or satellite has reached a certain altitude in the atmosphere, the atmospheric friction becomes negligible and the precessional motion remains such that the axis of precession makes a constant angle with the axis of the rocket.

To describe the motion of the rocket or satellite after its motion of precession becomes constant, let X, Y, Z be a right handed system of rectangular coordinate axes with the X-Y plane parallel to the equatorial plane of the Earth, the origin at any point of the trajectory or orbit of the moving rocket or satellite, and with the X axis pointing toward the vernal equinox.

Let i, j, k be a system of unit vectors parallel to the X, Y, Z axes respectively. Then the position vector from the origin (0, 0, 0) to x, y, z is given by

$$R = i\,x + j\,y + k\,z$$

if $\theta$ and $\Phi$ are the latitude and longitude respectively and r is the distance

$$R = r\,(i \cos\theta \cos\Phi + j \cos\theta \sin\Phi + k \sin\theta).$$

We may introduce a new system of base unit vectors $e_r$, $e_\theta$, and $e_\Phi$ by

$$\begin{aligned} e_r &= \frac{\delta R}{\delta r} = i \cos\theta \cos\Phi + j \cos\theta \sin\Phi + k \sin\theta \\ e_\theta &= \frac{1}{r}\frac{\delta R}{\delta\theta} = -\,i \sin\theta \cos\Phi - j \sin\theta \sin\Phi + k \cos\theta \qquad (1\text{-}1) \\ e_\Phi &= \frac{1}{r\cos\theta}\frac{\delta R}{\delta\Phi} = -\,i \sin\Phi + j \cos\Phi \end{aligned}$$

Let $e_r$ be parallel to the axis of precession of the rocket or satellite. Let $e_1'$, $e_2'$, and $e_r'$ be a third system of base unit vectors with $e_1'$ in the plane of $e_\theta$ and $e_\Phi$, $e_2'$ making an angle $\alpha$ with the $e_\theta e_\Phi$ plane and $e_r'$ parallel to the axis of the rocket or satellite. If $w_o$ is the angular velocity of rotation of the axis of the rocket or satellite about its axis of precession then,

$$e_1' = e_\theta \cos w_o t + e_\Phi \sin w_o t$$

$$e_2' = e_r \sin \alpha - e_\theta \cos \alpha \sin w_o t + e_\Phi \cos \alpha \cos w_o t$$

$$e_r' = e_r \cos \alpha + (e_\theta \sin w_o t - e_\Phi \cos w_o t) \sin \alpha.$$

To take into account the rotation of the rocket or satellite about its axis, let w be the angular velocity of rotation of the rocket or satellite about its axis, $e_r''$, $e_1''$, $e_2''$ a fourth system of base unit vectors with $e_r''$ parallel to $e_r'$ and with $e_1''$ and $e_2''$ in the plane of $e_1'$ and $e_2'$, then

$$e_r'' = e_r \cos \alpha + (e_\theta \sin w_o t - e_\Phi \cos w_o t) \sin \alpha$$

(2-1) $$e_1'' = e_r \sin \alpha \sin w\, t + e_\theta (\cos w_o t \cos w\, t - \cos \alpha \sin w_o t \sin w\, t) + e_\Phi (\sin w_o t \cos w\, t + \cos \alpha \cos w_o t \sin w\, t)$$

$$e_2'' = e_r \sin \alpha \cos w\, t - e_\theta (\cos w_o t \sin w\, t + \cos \alpha \sin w_o t \cos w\, t) - e_\Phi (\sin w_o t \sin w\, t - \cos \alpha \cos w_o t \cos w\, t).$$

Let $H_o$ be the magnitude of the Earth's magnetic field and M a unit vector along the Earth's field then

$$e_r'' H_r'' + e_1'' H_1'' + e_2'' H_2'' = M H_o.$$

Thus the scalar product of $e_r''$ with the vector $MH_o$ gives

$$H_r'' = H_o e_r'' \cdot M$$

$$\frac{\dot{H}_r''}{H_o} = e_r'' \cdot M'' \cos(\sphericalangle e_r'', M) = \cos \gamma_H (t)$$

$$\cos \gamma_H (t) = M \cdot e_r \cos \alpha + M \cdot e_\theta \sin \alpha \sin w_o t - M \cdot e_\Phi \sin \alpha \cos w_o t$$

If we set $M \cdot e_\theta = m$, $M \cdot e_\Phi = n$, $M \cdot e_r = L = \cos \beta_H$

$H_e''' = H_o (\cos \beta_H \cos \alpha + \sin \alpha (m \sin w_o t - n \cos w_o t))$, the critical values are given by $\frac{dH_r''}{dt} = 0$, $\tan w_o t = - \frac{m}{n}$

$$m^2 + n^2 = 1 - \cos^2 \beta_H = \sin^2 \beta_H.$$

(3-1) The minimum is given by $w_o t_o = \arcsin \frac{-m}{\sin \beta_H} = \arccos \frac{+n}{\sin \beta_H}$ and the maximum by $w_o t_1 = \arcsin \frac{m}{\sin \beta_H} = \arccos \frac{-n}{\sin \beta_H}$

Thus for the minimum

$$H_r'' = H_o (\cos \beta_H \cos \alpha - \sin \beta_H \sin \alpha) = H_o \cos (\beta_H + \alpha)$$

(3-2) and for the maximum

$$H_r'' = H_o (\cos \beta_H \cos \alpha + \sin \beta_H \sin \alpha) = H_o \cos (\beta_H - \alpha).$$

Since

$$\frac{H_r''}{H_o} = \cos (\sphericalangle e_r'', M) = \cos \gamma_H(t),$$

From (3-1) and (3-2)

$$\cos \gamma_H (t_o) = \cos (\beta_H + \alpha), \qquad \cos \gamma_H(t_1) = \cos (\beta_H - \alpha).$$

Writing

$$\gamma_o = \gamma_H (t_o), \qquad \gamma_1 = \gamma_H (t_1),$$

we find

$$\gamma_o = \beta_H + \alpha, \qquad \gamma_1 = \pm (\beta_H - \alpha)$$

$$\beta_H = \frac{\gamma_o \pm \gamma_1}{2} \qquad \alpha = \frac{\gamma_o \mp \gamma_1}{2},$$

where the upper signs are to be taken if $\beta_H > \alpha$ and the lower signs if $\beta_H < \alpha$.

Similarly if S is a unit vector parallel but in the opposite sense to the Sun's rays,

$$S = S_r'' e_r'' + S_1'' e_1'' + S_2'' e_2''$$

$$S \cdot e_r'' = \cos \gamma_S' (t) = S_r''.$$

A calculation identical to that in the preceeding section where S replaces H leads immediately to

(4-1) $$\beta_S = \frac{\gamma_{0S} \pm \gamma_{1S}}{2} \qquad \alpha = \frac{\gamma_{0S} \pm \gamma_{1S}}{2}$$

where the upper signs are to be taken if $\beta_S > \alpha$, and the lower signs are to be taken if $\beta_S < \alpha$,*

where

$$\gamma_{0S} = \gamma_S \ (t_3) = \arccos \ ( \ S_r'' \ (t_3))$$

$$\gamma_{1S} = \gamma_S \ (t_4) = \arccos \ ( \ S_r'' \ (t_4)).$$

However, here $\gamma_S(t_3)$ and $\gamma_S\ (t_4)$ are the actual measured angles between $e_r''$ and the Sun vector. Thus $\alpha$ and $\beta_S$ are uniquely determined. $\beta_H$ is thus uniquely determined by the formula

(4-2) $$\sin \beta_H = \frac{\cos_H (t_1) - \cos_H \ (t_0)}{2 \sin \alpha}$$

If the rocket or satellite has no precessional motion then

$$\alpha = 0, \qquad w_0 = 0, \qquad e_r'' = e_r$$

and

$$\cos \beta_H = \cos \gamma_H \ (t) = \text{constant}$$

$$\cos \beta_S = \cos \gamma_S \ (t) = \text{constant}$$

---

* If the period of rotation of the rocket about its axis, as determined by the magnetometers, is greater than that determined by the Sun sensors, then $\beta_H < \alpha$, $\beta_S > \alpha$, if the opposite is true then $\beta_S$ [illegible] $\alpha$, $\beta_H$ [illegible] $\alpha$.

## 2. CALCULATIONS OF $\theta$, AND $\Phi$, THE LATITUDE AND LONGITUDE OF THE AXIS OF PRECESSION

Let $\theta_H$ be the angle between the unit vector M and the equatorial plane, and $\Phi_H$ be the angle between the vertical plane containing M and the X-Z plane, then:

$$M = i \cos\theta_H \cos\Phi_H + j \cos\theta_H \sin\Phi_H + k \sin\theta_H.$$

Making use of the first of (1-1) we find

$$\cos\theta \cos\theta_H \cos(\Phi - \Phi_H) + \sin\theta_H \sin\theta = \cos\beta_H.$$

If $\theta_S$ and $\Phi_S$ are the corresponding angles for S, then:

$$S = i \cos\theta_S \cos\Phi_S + j \cos\theta_S \sin\Phi_S + k \sin\theta_S$$

$$\cos\theta \cos\theta_S \cos(\Phi - \Phi_S) + \sin\theta_S \sin\theta = \cos\theta_S.$$

Thus for the equations:

$$\cos\theta \cos\theta_H \cos(\Phi - \Phi_H) + \sin\theta_H \sin\theta = \cos\beta_H$$

$$\cos\theta \cos\theta_S \cos(\Phi - \Phi_S) + \sin\theta_S \sin\theta = \cos\beta_S.$$

Eliminating $\sin\theta$

$$\cos\theta = \frac{\cos\beta_H \sin\theta_S - \cos\beta_S \sin\theta_H}{\cos\theta_H \sin\theta_S \cos(\Phi-\Phi_H) - \cos\theta_S \sin\theta_H \cos(\Phi-\Phi_S)}$$

and eliminating $\cos\theta$

$$\sin\theta = \frac{\cos\beta_S \cos\theta_H \cos(\Phi-\Phi_H) - \cos\beta_H \cos\theta_S \cos(\Phi-\Phi_S)}{\cos\theta_H \sin\theta_H \cos(\Phi-\Phi_S) - \cos\theta_S \sin\theta_H \cos(\Phi-\Phi_S)}$$

If we let a, $b_1$, $b_2$, $c_1$, $c_2$, denote the following expressions:

$$a = \cos\beta_H \sin\theta_S - \cos\beta_S \sin\theta_H$$

$$b_1 = \cos\theta_H \sin\theta_S \cos\Phi_H - \cos\theta_S \sin\theta_H \cos\Phi_S$$

$$b_2 = \cos\theta_H \sin\theta_S \sin\Phi_H - \cos\theta_S \sin\theta_H \sin\Phi_S$$

$$c_1 = \cos\theta_H \cos\beta_S \cos\Phi_H - \cos\theta_S \cos\beta_H \cos\Phi_S$$

$$c_2 = \cos\theta_H \cos\beta_S \sin\Phi_H - \cos\theta_S \cos\beta_H \sin\Phi_S,$$

$$\cos\theta = \frac{a}{b_1 \cos\Phi + b_2 \sin\Phi}$$

(6-1)

$$\sin\theta = \frac{c_1 \cos\Phi + c_2 \sin\Phi}{b_1 \cos\Phi + b_2 \sin\Phi}$$

The equations (6-1) may be written

$$b_1 \cos\theta \cos\Phi + b_2 \cos\theta \sin\Phi = a$$

$$(b_1 \sin\theta - c_1) \cos\Phi + (b_2 \sin\theta - c_2) \sin\Phi = 0$$

The solution of these equations is

$$\sin\Phi = \frac{a}{\cos\theta} \frac{b_1 \sin\theta - c_1}{b_1 c_2 - b_2 c_1}$$

(6-2)

$$\cos\Phi = -\frac{a}{\cos\theta} \frac{b_2 \sin\theta - c_2}{b_1 c_2 - b_2 c_1}$$

$$\tan\Phi = -\frac{b_1 \sin\theta - c_1}{b_2 \sin\theta - c_2}$$

Eliminating $\Phi$ from the equation (6-2)

(6-3) $$\sin\theta = \frac{a^2(b_1c_1 + b_2c_2) \pm (b_1c_2 - b_2c_1)\sqrt{(b_1c_2 - b_2c_1)^2 + a^2(b_1^2 + b_2^2 - c_1^2 - c_2^2}}{(b_1^2 + b_2^2)a^2 + (b_1c_2 - b_2c_1)^2}$$

Substituting for a, $b_1$, $b_2$, $c_1$, $c_2$ we find

$$(b_1c_2 - b_2c_1)^2 + a^2(b_1^2 + b_2^2 - c_1^2 - c_2^2 - a^2) = a^2 \{\sin^2\beta_S \sin^2\beta_H - (M \cdot S - \cos\beta$$

$$b_1c_1 + b_2c_2 = \sin\theta_S \cos\beta_S + \sin\theta_H \cos\beta_H - M \cdot S(\cos\beta_H \sin\theta_S + \cos\beta_S \sin\theta$$

$$b_1c_2 - b_2c_1 = a \cos\theta_H \cos\theta_S \sin(\Phi_H - \Phi_S)$$

$$(b_1c_2 - b_2c_1)^2 + (b_1^2 + b_2^2)a^2 = a^2\{1 - (M \cdot S)^2\}$$

Thus (6-3) may be written

(7-1)
$$\sin\theta = \frac{\sin\theta_S\cos\beta_S + \sin\theta_H\cos\beta_H - M\cdot S\,(\cos\beta_H\sin\theta_S + \cos\beta_S\sin\theta_H)}{1-(M\cdot S)^2}$$
$$\pm \frac{\cos\theta_H\cos\theta_S\sin(\Phi_H-\Phi_S)\sqrt{\sin^2\beta_S\sin^2\beta_H - (M\cdot S - \cos\beta_S\cos\beta_H)^2}}{1-(M\cdot S)^2}$$

With this determination of $\Phi$ and $\theta$ the base vectors $e_\theta$, $e_\Phi$, and $e_r$ in (1-1)

(7-2)
$$e_\theta = -i\sin\theta\cos\Phi - j\sin\theta\sin\Phi + k\cos\Phi$$
$$e_\Phi = -i\sin\Phi + j\cos\Phi$$
$$e_r = i\cos\theta\cos\Phi + j\cos\theta\sin\Phi + k\sin\theta$$

are fully determined, and so also

$M\cdot e_r$, $M\cdot e_\theta$, and $M\cdot e_\Phi$, i.e. $\cos\beta_H$, $m$, $n$, $S\cdot e_r$, $S\cdot e_\theta$, $S\cdot e_\Phi$, $\cos\beta_S$, $m_S$, $n_S$,

Now from (2-1), when $\alpha \neq 0$

(7-3)
$$e_r'' = e_r\cos\alpha + (e_\theta\sin w_o t - e_\Phi\cos w_o t)\sin\alpha$$

and $e_r''$ is determined for any t, however this last expression does not specify when t is to be counted zero. It is convenient to start counting the time at a time when $S_r''$ is a minimum, i.e. according to (3-1).

(7-4)
$$w_o t_3 = \arcsin\frac{-m_S}{\sin\beta_S} = \arccos\frac{+n_S}{\sin\beta_S}$$

Let t be defined

$$t = t_3 + T$$
$$w_o t = w_o t_3 + w_o T$$

---

* see Appendix A for the sign before the radical. pp 13-14

$$\sin w_o t = \sin (w_o t_3 + w_o T) + \sin w_o t_3 \cos w_o T + \cos w_o t_3 \sin w_o T$$

$$\cos w_o t + \cos (w_o T_3 + w_o T) = \cos w_o t_3 \cos w_o T - \sin w_o t_3 \sin w_o T.$$

Substituting (7-4) in these two equations we find

$$\text{(8-1)} \qquad \sin w_o t = \frac{n_S \sin w_o T - m_S \cos w_o T}{\sin \beta_S}$$

$$\cos w_o t = \frac{n_S \cos w_o T + m_S \sin w_o T}{\sin \beta_S}$$

Substituting (8-1) in (7-3)

$$\text{(8-2)} \qquad e_r'' = e_r \cos \alpha + \left[ e_\theta \frac{n_S \sin w_o T - m_S \cos w_o T}{\sin \beta_S} - e_\Phi \frac{n_S \cos w_o T + m_S \sin w_o T}{\sin \beta_S} \right] \sin \alpha.$$

## 3. ASPECT WITH RESPECT TO THE ROTATING EARTH

In order to find the aspect of the rocket or satellite with respect to the Earth based system of coordinates axes, let $w_e$ be the angular velocity of rotation of the Earth about its axis, t the time in seconds measured from midnight December 31 - January 1. Then if $I_1$, $I_2$ and $I_3$ are a system of orthonormal base vectors parallel to the X', Y', Z' axes respectively, with X' and Y' in the equatorial plane of the Earth, and the X' axis in the Greenwich Meridian plane, and $w_e t$ measured from the X axis or from the base vector i, we have

$$I_1 = i \cos (w_e t - \delta) + j \sin (w_e t - \delta)$$

$$I_2 = - i \sin (w_e t - \delta) + j \cos (w_e t - \delta)$$

$$I_3 = k$$

and

$$i = I_1 \cos (w_e t - \delta) - I_2 \sin (w_e t - \delta)$$

$$j = I_1 \sin (w_e t - \delta) + I_2 \cos (w_e t - \delta)$$

$$k = I_3$$

where $\delta$ is the angle between the vectors i and $I_1$ measured clockwise from i to $I_1$ at midnight December 31 - January 1. Substituting the relations above in (7-2) i.e. in

$$e_r = i \cos \theta \cos \Phi + j \cos \theta \sin \Phi + k \sin \theta$$

$$e_\theta = - i \sin \theta \cos \Phi - j \sin \theta \sin \Phi + k \cos \theta$$

$$e_\Phi = - i \sin \Phi + j \cos \Phi,$$

we find

(9-1)
$$\begin{aligned} e_r &= I_1 \cos \theta \cos (w_e t - \Phi - \delta) - I_2 \cos \theta \sin (w_e t - \Phi - \delta) + I_3 \sin \theta \\ e_\theta &= - I_1 \sin \theta \cos (w_e t - \Phi - \delta) + I_2 \sin \theta \sin (w_e t - \Phi - \delta) + I_3 \cos \theta \\ e_\Phi &= I_1 \sin (w_e t - \Phi - \delta) + I_2 \cos (w_e t - \Phi - \delta). \end{aligned}$$

Now (7-3)

(9-2)
$$e_r'' = e_r \cos \alpha + (e_\theta \sin w_o t - e_\Phi \cos w_o t) \sin \alpha.$$

Again let $t_3$ be the time corresponding to the minimum angle for the first Sun fix, then

$$t = t_3 + T.$$

and according to (8-1) we find

$$\sin w_o t = \frac{n_S \sin w_o T - m_S \cos w_o T}{\sin \beta_S}$$

$$\cos w_o t = \frac{n_S \cos w_o T + m_S \sin w_o T}{\sin \beta_S}$$

To simplify these expressions let $\gamma$ be defined by

$$\tan \gamma = \frac{m_S}{n_S} \qquad m_S^2 + n_S^2 = \sin^2 \beta_S$$

$$m_S = \sin \beta_S \sin \gamma, \qquad n_S = \sin \beta_S \cos \gamma$$

then,

$$\sin w_o t = \sin (w_o T - \gamma), \qquad \cos w_o t = \cos (w_o T - \gamma).$$

In order to simplify the expression (9-1) and (9-2), let

$$\phi = w_e t - \Phi - \delta$$

then the equations (9-1) become,

(10-1)
$$e_r = I_1 \cos \phi \cos \theta - I_2 \sin \phi \cos \theta + I_3 \sin \theta$$
$$e_\theta = - I_1 \cos \phi \sin \theta + I_2 \sin \phi \sin \theta + I_3 \cos \theta$$
$$e_\Phi = I_1 \sin \phi + I_2 \cos \phi$$

and (9-2) becomes

$$e_r'' = e_r \cos \alpha + \left\{ e_\theta \sin (w_o T - \gamma) - e_\Phi \cos (w_o T - \gamma) \right\} \sin \alpha$$

The expression for $e_r''$ in terms of $I_1$, $I_2$, and $I_3$ becomes

(10-2)
$$e_r'' = I_1 \Big[ \cos \phi \left\{ \cos \theta \cos \alpha - \sin \theta \sin \alpha \sin (w_o T - \gamma) \right\} - \sin \phi \sin \alpha \cos(w_o T -$$
$$- I_2 \Big[ \sin \phi \left\{ \cos \theta \cos \alpha - \sin \theta \sin \alpha \sin (w_o T - \gamma) \right\} + \cos \phi \sin \alpha \cos(w_o T -$$
$$+ I_3 \Big[ \sin \theta \cos \alpha + \cos \theta \sin \alpha \sin (w_o T - \gamma) \Big].$$

If the rocket or satellite has no precessional motion $\alpha = 0$ and,

$$e_r'' = I_1 \cos \phi \cos \theta - I_2 \sin \phi \cos \theta + I_3 \sin \theta.$$

The expression (10-2) is the representation of the unit vector parallel to the rocket or satellite axis. Thus if V is any vector with components $(V_1, V_2, V_3)$ in the base $I_1$, $I_2$, and $I_3$ i.e. if

$$V = I_1V_1 + I_2V_2 + I_3V_3,$$

the component of V along $e_r''$ is given by $e_r'' \cdot V = V_r''$ , i.e.

$$\begin{aligned} V_r'' = & V_1\Big[\cos\phi\,\big\{\cos\theta\cos\alpha - \sin\theta\sin\alpha\sin(w_oT-\gamma)\big\} - \sin\phi\sin\alpha\cos(w_oT-\gamma)\Big] \\ & - V_2\Big[\sin\phi\,\big\{\cos\theta\cos\alpha - \sin\theta\sin\alpha\sin(w_oT-\gamma)\big\} + \cos\phi\sin\alpha\cos(w_oT-\gamma)\Big] \\ & + V_3\Big[\sin\theta\cos\alpha + \cos\theta\sin\alpha\sin(w_oT-\gamma)\Big] \;. \end{aligned}$$

If the rocket or satellite has a Sun sensor mounted on it such that the axis of the sensor makes an angle of 116° with the rocket or satellite axis and if $\pi$ is the angle which the Sun vector S makes with the axis of the sensor, the relation between the angle $\gamma_S(t) = (\measuredangle S, e_r'')$ and $\pi$ is given by

$$\gamma_S(t) = 116^\circ - \pi(t)$$

(11-1)
$$\frac{d\gamma_S(t)}{dt} = -\frac{d\pi}{dt} \;.$$

The critical values of $\gamma_S(t)$ are given by

$$\frac{d\gamma_S(t)}{dt} = 0$$

and therefore also by

$$\frac{d\pi}{dt} = 0,$$

and

$$\frac{d^2\gamma_S(t)}{dt^2} = -\frac{d^2\pi}{dt^2} \,.$$

Thus for the maximum values of $\gamma_S(t)$, $\frac{d^2 \gamma_S(t)}{dt} < 0$ and $\frac{d^2 \pi(t)}{dt} > 0$ therefore, $\pi(t)$ is maximum when $\gamma_S(t)$ is minimum and $\pi(t)$ is minimum when $\gamma_S(t)$ is maximum. Now we have seen on page 4 that maximum $\gamma_S(t_3) = \pm(\beta_S - \alpha)$ and minimum $\gamma_S(t_4) = \beta_S - \alpha$. Thus from (12-1)

(12-1)
$$\pi(t) = 116^0 - \gamma_S(t)$$
$$\pi(t_3) = 116^0 - \beta_S - \alpha$$
$$\pi(t_4) = 116^0 \pm (\beta_S - \alpha).$$

Thus if $\beta_S$ 0

$$116^o - \beta_S = \frac{\pi(t_4) + \pi(t_3)}{2}$$
$$\alpha = \frac{\pi(t_4) - \pi(t_3)}{2}$$

and

(12-2)
$$\beta_S = 116^o - \frac{\pi(t_4) + \pi(t_3)}{2}$$
$$\alpha = \frac{\pi(t_4) - \pi(t_3)}{2}$$

If $\beta_S$ $\alpha$

$$\beta_S = \frac{\pi(t_4) - \pi(t_3)}{2}$$
$$\alpha = 116^o - \frac{\pi(t_4) + \pi(t_3)}{2}$$

The Adcole Sun Sensor, manufactured by the Adcole Research Corporation, is designed so that $\pi < 0$ for $116^o < \gamma_S(t) < 180^o$, $\pi > 0$ $52^o < \gamma_S(t) < 116^o$.

## APPENDIX A

### DETERMINATION OF THE SIGN IN THE EXPRESSION FOR SIN $\theta$

If $e_{r_1}$ and $e_{r_2}$ lie in a plane perpendicular to the M-S plane and make equal angles with that plane, with $e_{r_1}$ on one side and $e_{r_2}$ on the other side of the M-S plane, then

$$e_{r_1} x (MxS) = e_{r_1} x (MxS).$$

Thus

$$(e_{r_1} \cdot S)M - (e_{r_1} \cdot M)S = (e_{r_2} \cdot S)M - (e_{r_2} \cdot M)S$$

and

$$\cos \beta_H = e_{r_1} \cdot M = e_{r_2} \cdot M, \qquad \cos \beta_S = e_{r_1} \cdot S = e_{r_2} \cdot S.$$

One root of sin $\theta$ corresponds to $e_r$ on one side of the M-S plane and the other root corresponds to $e_r$ on the other side of the M-S plane. Now it is easy to show that the triple scalar product of M, S, $e_r$

$$(MSe_r) = \pm\sqrt{\sin^2 \beta_S \sin^2 \beta_H - (M \cdot S - \cos \beta_S \cos \beta_H)^2}.$$

Obviously if the angle between MxS and $e_r$ is less than 90° the + sign has to be taken for (M, S, $e_r$) and the - sign if $e_r$ makes an angle greater than 90°.

Let $\gamma_{H_o}$ and $\gamma_{H_1}$ correspond to the maximum and minimum angles between the axis of the rocket or satellite and the magnetic field respectively, and $\gamma_{S_2}$ and $\gamma_{S_3}$ the corresponding angles between the Sun vector and the rocket or satellite axis respectively. If the rocket is precessing in a clockwise direction with respect to an observer on the ground, then in the records of $\gamma_H$ and $\gamma_S$ transmitted to the ground the following sequences will be observed for $\gamma_{H_o}$, $\gamma_{H_1}$, $\gamma_{S_2}$, $\gamma_{S_3}$ :

If $e_r$ makes an angle of less than 90° with MxS the sequence of maximum and minimum angles will be as follows:

$$\gamma_{S_3}\,,\ \gamma_{H_0}\,,\ \gamma_{S_2}\,,\ \gamma_{H_1}\ ,$$

that is, if this sequence occurs in a given flight the sign + must be taken. If $e_r$ makes an angle greater than 90°with MxS, the sequence of maximum and minimum will be as follows:

$$\gamma_{S_3}\,,\ \gamma_{H_1}\,,\ \gamma_{S_2}\,,\ \gamma_{H_0}\ ,$$

in this case then, the negative sign before the radical must be taken. The system breaks down only if $(MSe_r) = 0$, but in this case there is only one root for sin $\theta$, namely,

$$\sin\theta = \frac{\sin\theta_S\cos\beta_S + \sin\theta_H\cos\beta_H - M\cdot S(\cos\beta_H\sin\theta_S + \cos\beta_S\sin\theta_H)}{1 - (M\cdot S)^2}\ .$$

AF Cambridge Research Laboratories, Bedford Mass. Geophysics Research Directorate.
ON DETERMINATION OF THE ORIENTATION OF THE AXIS OF A ROCKET OR SATELLITE IN ITS TRAJECTORY OR ORBIT. by R.J. Marcou, October 1963. 14 pp. AFCRL -63-871
Unclassified report.

A method is developed for determining the aspect of the axis of a rocket or satellite with respect to an earth based system of coordinates for the case where these bodies undergo a constant precessional motion about some fixed direction. The analysis is based on data obtained from a magnetometer mounted on the body so as to give the axial component of the Earth's magnetic field and a sun sensor which measures the angle between the Sun vector and the axis of the body.

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